SAMPLE PAPER

issued by CBSE for Board Exams (2025-26)

Mathematics (041) - Class 12

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (with a total of **4 marks** for each Case-study / Source-based / Passage-based question).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

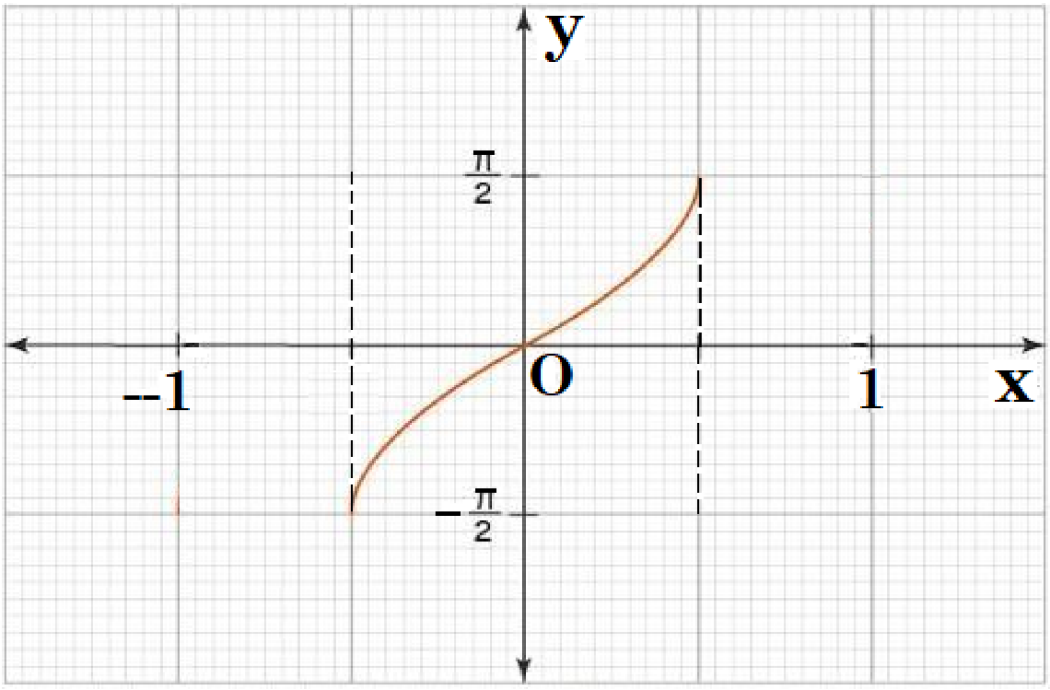
   You have to attempt only one of the alternatives in all such questions.
4. Use of calculator is not allowed.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*

*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. Identify the function shown in the graph.



(a) $\sin^{-1} x$ (b) $\sin^{-1}(2x)$ (c) $\sin^{-1}\left(\frac{x}{2}\right)$ (d) $2\sin^{-1} x$

01. **FOR VISUALLY IMPAIRED**

Inverse trigonometric function, whose domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$, is

(a) $\cos^{-1} x$ (b) $\cos^{-1}\left(\frac{x}{3}\right)$ (c) $\cos^{-1}(3x)$ (d) $3\cos^{-1} x$

02. If for three matrices $A=[a_{ij}]_{m\times 4}$, $B=[b_{ij}]_{n\times 3}$ and $C=[c_{ij}]_{p\times q}$, the products AB and AC both are defined and are square matrices of same order, then the values of m, n, p and q are

(a) $m=q=3$ and $n=p=4$ (b) $m=2$, $q=3$ and $n=p=4$

(c) $m=q=4$ and $n=p=3$ (d) $m=4$, $p=2$ and $n=q=3$

03. If the matrix $A=\begin{bmatrix} 0 & r & -2 \\ 3 & p & t \\ q & -4 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $\frac{q+t}{p+r}$ is

(a) -2 (b) 0 (c) 1 (d) 2

04. If A is a square matrix of order 4 and $|\text{adj}\,A| = 27$, then $A(\text{adj}\,A)$ is equal to

(a) 3 (b) 9 (c) 3I (d) 9I

05. The inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is

(a) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (c) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

06. Value of the determinant $\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix}$ is

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

07. If a function defined by $f(x)=\begin{cases} kx+1, & x \le \pi \\ \cos x, & x > \pi \end{cases}$ is continuous at $x=\pi$, then the value of k is

(a) π (b) $-\frac{1}{\pi}$ (c) 0 (d) $-\frac{2}{\pi}$

08. If $f(x) = x\tan^{-1} x$, then $f'(1)$ is equal to

(a) $\frac{\pi}{4}-\frac{1}{2}$ (b) $\frac{\pi}{4}+\frac{1}{2}$ (c) $-\frac{\pi}{4}-\frac{1}{2}$ (d) $-\frac{\pi}{4}+\frac{1}{2}$

09. A function $f(x)=10-x-2x^2$ is increasing on the interval

(a) $\left(-\infty, -\frac{1}{4}\right]$ (b) $\left(-\infty, \frac{1}{4}\right)$ (c) $\left[-\frac{1}{4}, \infty\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

10. The solution of the differential equation $x\,dx + y\,dy = 0$ represents a family of

(a) straight lines (b) parabolas (c) circles (d) ellipses

11. If $f(a+b-x)=f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ (b) $\frac{a+b}{2}\int_a^b f(a-x)\,dx$

(c) $\frac{b-a}{2}\int_a^b f(x)\,dx$ (d) $\frac{a+b}{2}\int_a^b f(x)\,dx$

12. If $\int x^3\sin^4(x^4)\cos(x^4)\,dx = a\sin^5(x^4)+C$, then a is equal to

(a) $-\frac{1}{10}$ (b) $\frac{1}{20}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

13. A bird flies through a distance in a straight line given by the vector $\hat{i}+2\hat{j}+\hat{k}$. A man standing beside a straight metro rail track given by $\vec{r}=(3+\lambda)\hat{i}+(2\lambda-1)\hat{j}+3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is

(a) $\frac{6}{\sqrt{14}}$ units (b) $\frac{14}{\sqrt{6}}$ units (c) $\frac{8}{\sqrt{14}}$ units (d) $\frac{5}{\sqrt{6}}$ units

14. The distance of the point with position vector $3\hat{i}+4\hat{j}+5\hat{k}$ from the y-axis is

(a) 4 units (b) $\sqrt{34}$ units (c) 5 units (d) $5\sqrt{2}$ units

15. If $\vec{a}=3\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-3\hat{k}$ and $\vec{c}=6\hat{i}-\hat{j}+2\hat{k}$ are three given vectors, then $(2\vec{a}.\hat{i})\hat{i}-(\vec{b}.\hat{j})\hat{j}+(\vec{c}.\hat{k})\hat{k}$ is same as the vector

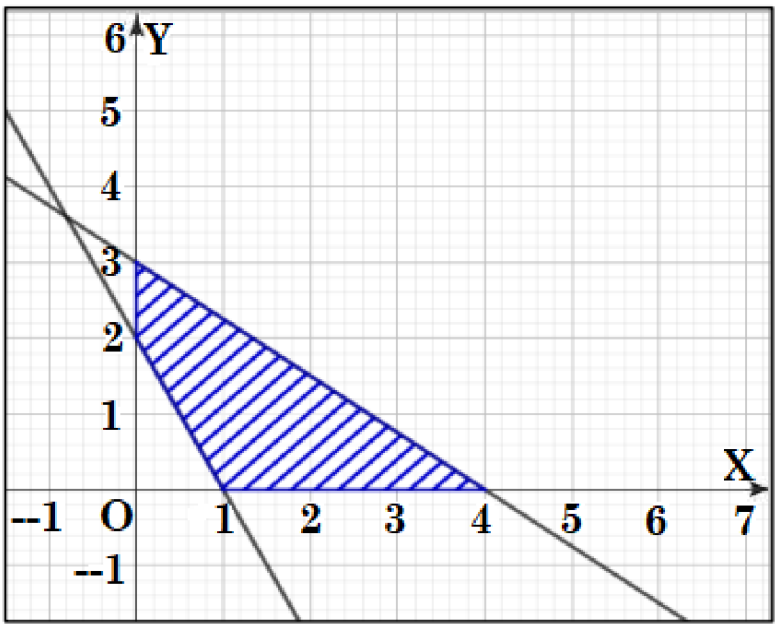
(a) $\vec{a}$ (b) $\vec{b}+\vec{c}$ (c) $\vec{a}-\vec{b}$ (d) $\vec{c}$

16. A student of class XII studying Mathematics comes across an incomplete question in a book.

Maximize $Z=3x+2y+1$

Subject to the constraints $x\ge 0$, $y\ge 0$, $3x+4y\le 12$.

S/he notices the below shown graph for the said LPP, and finds that a constraint is missing in it. Help her/him choose the required constraint from the graph.



The missing constraint is

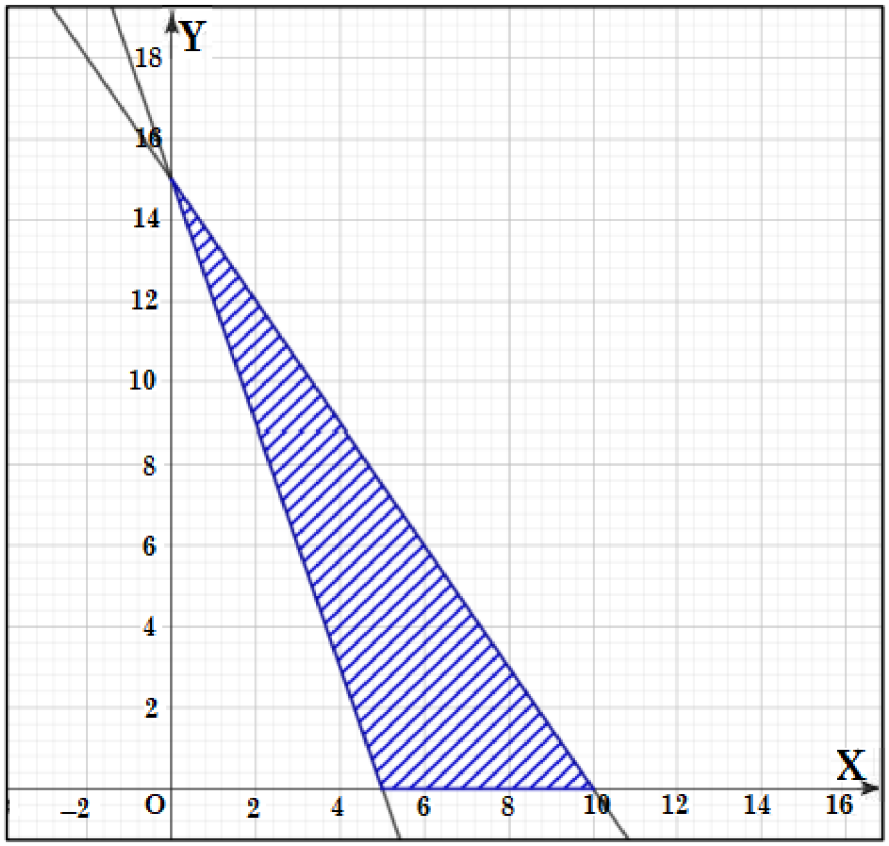
(a) $x+2y\le 2$ (b) $2x+y\ge 2$ (c) $2x+y\le 2$ (d) $x+2y\ge 2$

16. **FOR VISUALLY IMPAIRED**

If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points $(4, 0)$ and $(0, 3)$ of the feasible region determined by the system of linear inequalities, then

(a) $4a = 3b$ (b) $3a = 4b$ (c) $4a + c = 3b$ (d) $3a + c = 4b$

17. The feasible region of a linear programming problem is bounded but the objective function attains its minimum value at more than one point. One of the points is $(5, 0)$.



Then one of the other possible points at which the objective function attains its minimum value is

(a) $(2, 9)$ (b) $(6, 6)$ (c) $(4, 7)$ (d) $(0, 0)$

17. **FOR VISUALLY IMPAIRED**

The graph of the inequality $3x + 5y < 10$ is the

(a) entire XY-plane

(b) open half plane that doesn't contain origin

(c) open half plane that contains origin, but not the points of the line $3x + 5y = 10$

(d) half plane that contains origin and the points of the line $3x + 5y = 10$

18. A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN?

(a) $\frac{1}{81}$ (b) $\frac{1}{100}$ (c) $\frac{1}{90}$ (d) 1

Followings are ***Assertion-Reason based questions.***

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A) :** Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\tan^{-1}1-\sec^{-1}(\sqrt{2})$ is $\frac{\pi}{4}$.

Reason (R) : Principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and that of $\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A) :** Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r}\times(\vec{a}+\vec{b})=\vec{0}$, then $\vec{r}$ is perpendicular to $\vec{a}\times\vec{b}$.

Reason (R) : The vector $(\vec{a}+\vec{b})$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$.

SECTION B

*(Question numbers 21 to 25 carry **2 marks** each.)*

21. (A) Evaluate $\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)$.

OR

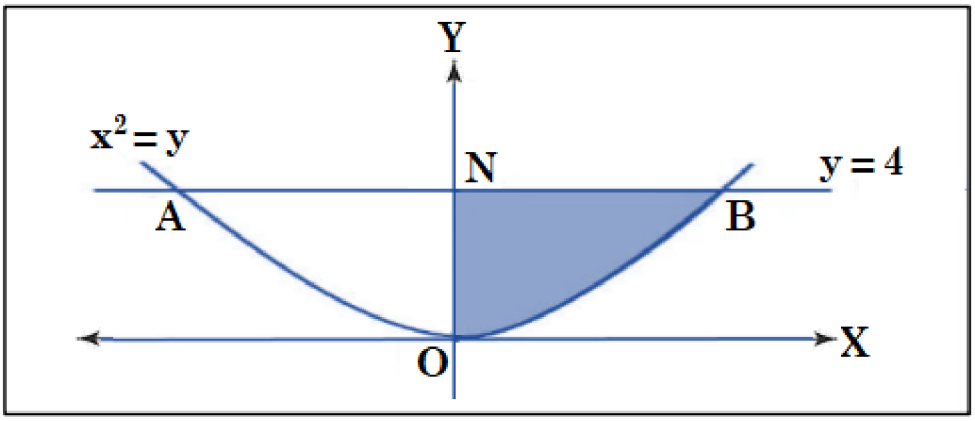
(B) Find the domain of $\cos^{-1}(3x-2)$.

22. If $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then prove that $\frac{dy}{dx}-\sec x=0$.

23. (A) Find : $\int\frac{(x-3)}{(x-1)^3}e^x dx$.

OR

(B) Find out the area of shaded region in the enclosed figure.



23. **FOR VISUALLY IMPAIRED**

(B) Find out the area of the region enclosed by the curve $y^2=x$, $x=3$ and x-axis in the first quadrant.

24. If $f(x+y)=f(x)\,f(y)$ for all $x, y\in\mathbb{R}$ and $f(5)=2$, $f'(0)=3$, then using the definition of derivatives, find $f'(5)$.

25. The two vectors $\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+3\hat{k}$ represent the two sides OA and OB, respectively of a ΔOAB, where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. (A) If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$ and hence find its value at $x=e$.

OR

(B) If $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$, find $\dfrac{d^2y}{dx^2}$.

27. A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.

28. (A) Sketch the graph $y = |x+1|$. Evaluate $\int_{-4}^{2} |x+1|\,dx$. What does the value of this integral represent on the graph?

OR

(B) Using integration find the area of the region $\{(x, y) : x^2 - 4y \le 0,\ y - x \le 0\}$.

28. **FOR VISUALLY IMPAIRED**

(A) Define the function $y = |x+1|$. Evaluate $\int_{-4}^{2} |x+1|\,dx$ What does the value of this integral represent?

OR

(B) Using integration find the area enclosed within the curve : $25x^2 + 16y^2 = 400$.

29. (A) Find the distance of point (2, –1, 3) from the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \mu(3\hat{i} + 6\hat{j} + 2\hat{k})$ measured parallel to the z-axis.

OR

(B) Find the point of intersection of the line $\vec{r} = (3\hat{i} + \hat{k}) + \mu(\hat{i} + \hat{j} + \hat{k})$ and the line passing through the point (2, –1, 1) parallel to the z-axis. How far is this point from the z-axis?

30. Solve the linear programming problem (L.P.P.) graphically.

Maximize $Z = 2x + y$

subject to $x + y \le 1200,\ x + y \ge 600,\ y \le \dfrac{x}{2},\ x \ge 0,\ y \ge 0$.

30. **FOR VISUALLY IMPAIRED**

The objective function $Z = 3x + 2y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are A(600, 0), B(1200, 0), C(800, 400) and D(400, 200). Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z.

31. Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. For two matrices $A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, find the product AB and hence solve the system of equations : $3x - 6y - z = 3,\ 2x - 5y - z + 2 = 0,\ -2x + 4y + z = 5$.

33. (A) Evaluate : $\int_0^1 \dfrac{\log(1+x)}{1+x^2}\,dx$.

OR

(B) Find : $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$.

34. (A) Solve the differential equation : $y + \frac{d}{dx}(xy) = x(\sin x + x)$.

OR

(B) Find the particular solution of the differential equation :

$$2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0 \text{ given that } y(0) = 1.$$

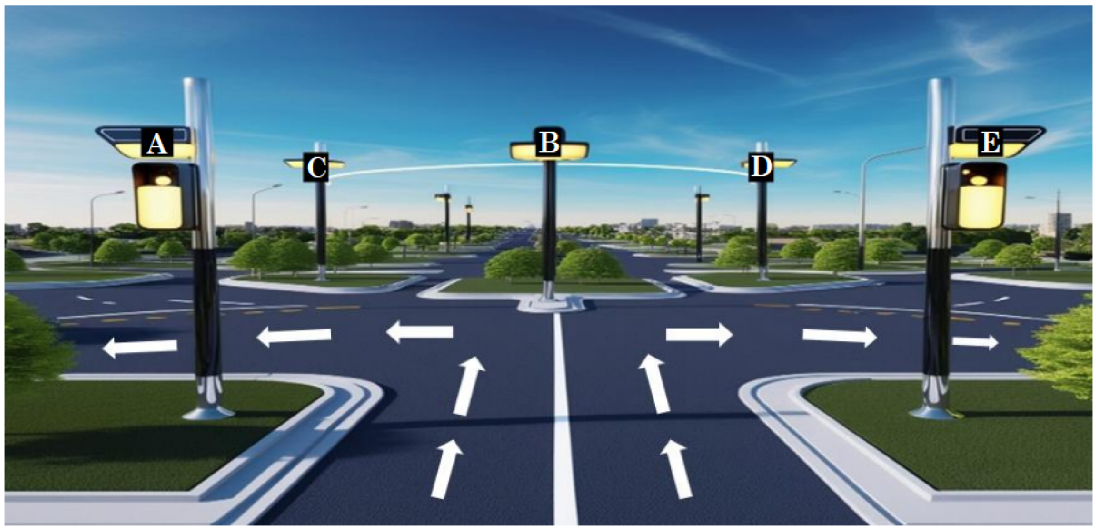
35. The lines $\frac{x-1}{3} = -y,\ z+1 = 0$ and $\frac{-x}{2} = \frac{y+1}{2} = z+2$ intersect at a point whose y-coordinate is 1. Find the coordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection.

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** (i) and (ii) of **2 marks** each.*

36. CASE STUDY I :

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals.
The city has five major spots labeled A, B, C, D and E.



The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions.

(i) Is the traffic flow reflexive? Justify.
(ii) Is the traffic flow transitive? Justify.
(iii) (A) Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

(iii) (B) Does the traffic flow represent a function? Justify your answer.

37. CASE STUDY II :

LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving the resources and money over time.

A company manufactures a new type of energy-efficient LED bulbs. The cost of production and the revenue generated by selling x bulbs (in an hour) are modeled as : $C(x) = 0.5x^2 - 10x + 150$ and $R(x) = -0.3x^2 + 20x$ respectively, where $C(x)$ and $R(x)$ are both in ₹.



To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions.

(i) Derive the profit function $P(x)$.

(ii) Find the critical points of $P(x)$.

(iii) (A) Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test.

OR

(iii) (B) Identify the possible practical value of x (i.e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit.

38. CASE STUDY III :

Excessive use of screens can result in vision problems, obesity, sleep disorders, anxiety, low retention problems and can impede social and emotional comprehension and expression. It is essential to be mindful of the amount of time we spend on screen and to reduce our screen time by taking regular breaks, setting time limits, and engaging in non-screen based activities.



In a class of students of the age group 14 to 17, the students were categorized into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours/day on the same activities. The third group spent less than 2 hours/day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group and 30% of third group having the same symptoms.

(i) What is the total percentage of students who suffer from anxiety and low retention issues in the class?

(ii) A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that s/he spends screen time more than 4 hours per day?



This paper has been issued by **CBSE** for **2025-26 Board Exams of class 12 Mathematics (041)**.

Note : We have **re-typed** the Official CBSE Sample Paper and, also added the **necessary illustrations** at some places.

If you notice any error which could have gone un-noticed, please do inform us via **message** on the **WhatsApp @ +919650350480** or, via **Email** at **iMathematicia@gmail.com**

Let's learn **Math** with **smile**:-)

 # O.P. GUPTA, Math Mentor

Author - MATHMISSION Series

🕮 Detailed Solutions for CBSE Sample Paper (2025-26)

SECTION A

01. (b) Clearly from the graph Domain is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. So graph is of the function $\sin^{-1}(2x)$.

01. **FOR VISUALLY IMPAIRED**

(c) Since the domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$. So the function is $\cos^{-1}(3x)$.

02. (a) AB is defined so, $n = 4$.
AC is defined so, $p = 4$.
AB and AC are square matrices of same order, so $m \times 3 = m \times q \Rightarrow q = 3 = m$.

03. (a) As A is skew symmetric so $p = 0,\ q = 2,\ r = -3,\ t = 4$.

Hence, $\frac{q+t}{p+r} = \frac{6}{-3} = -2$.

04. (c) Given $|\text{adj } A| = 27$

$\Rightarrow |A|^{4-1} = 27$

$\Rightarrow |A|^3 = 3^3$

$\Rightarrow |A| = 3$

Hence, $A(\text{adj } A) = |A| I = 3I$.

05. (b) Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$.

Since the inverse of diagonal matrix is given by taking reciprocal of all of its diagonal elements.

06. (a) $\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix} = \cos 67^\circ \cos 23^\circ - \sin 67^\circ \sin 23^\circ = \cos(67^\circ + 23^\circ) = \cos 90^\circ = 0$.

07. (d) $f(x)$ is continuous at $x = \pi$

$\Rightarrow \lim\limits_{x \to \pi^-} (kx + 1) = \lim\limits_{x \to \pi^+} \cos x = f(\pi)$

$\Rightarrow \lim\limits_{h \to 0} [k(\pi - h) + 1] = \lim\limits_{h \to 0} \cos(\pi + h) = k\pi + 1$

$\Rightarrow k\pi + 1 = -1$ [$\because \cos \pi = -1$

$\Rightarrow k = -\frac{2}{\pi}$.

08. (b) $f(x) = x \tan^{-1} x$

$\Rightarrow f'(x) = 1 \times \tan^{-1} x + x \times \frac{1}{1+x^2} = \tan^{-1} x + \frac{x}{1+x^2}$

$\therefore f'(1) = \tan^{-1} 1 + \frac{1}{1+1^2} = \frac{\pi}{4} + \frac{1}{2}$.

09. (a) For the given function $f(x) = 10 - x - 2x^2$, $f'(x) = -1 - 4x$.
For increasing function, $f'(x) \geq 0$

$\Rightarrow -(1 + 4x) \geq 0$

$\Rightarrow (1 + 4x) \leq 0$

$$\Rightarrow x \le -\frac{1}{4}$$

$$\therefore x \in \left(-\infty, -\frac{1}{4}\right].$$

10. (c) $x\,dx + y\,dy = 0$

$\Rightarrow \int x\,dx = -\int y\,dy$

$\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$

$\Rightarrow x^2 + y^2 = 2k$

Clearly, $x^2 + y^2 = 2k$ represents a family of circles.

11. (d) Let $I = \int_a^b x\,f(x)\,dx = \int_a^b (a+b-x)\,f(a+b-x)\,dx$

$\Rightarrow I = \int_a^b (a+b-x)\,f(x)\,dx$ [Given $f(a+b-x) = f(x)$

$\Rightarrow I = \int_a^b (a+b)\,f(x)\,dx - \int_a^b x\,f(x)\,dx$

$\Rightarrow 2I = (a+b)\int_a^b f(x)\,dx$

$\therefore I = \frac{1}{2}(a+b)\int_a^b f(x)\,dx$.

12. (b) Let $I = \int x^3 \sin^4(x^4)\cos(x^4)\,dx$

Put $\sin(x^4) = t \Rightarrow 4x^3\cos(x^4)\,dx = dt \Rightarrow x^3\cos(x^4)dx = \frac{1}{4}dt$.

Thus $I = \frac{1}{4}\int t^4\,dt = \frac{1}{20}t^5 + C = \frac{1}{20}\sin^5(x^4) + C$

That is, $I = \frac{1}{20}\sin^5(x^4) + C = a\sin^5(x^4) + C$

On comparing, we get $a = \frac{1}{20}$.

13. (c) The projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the line $\vec{r} = (3\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is given by $\vec{p}.\hat{q} = \vec{p}.\left(\frac{\vec{q}}{|\vec{q}|}\right) = \frac{1\times1 + 2\times2 + 1\times3}{\sqrt{1^2+2^2+3^2}} = \frac{8}{\sqrt{14}}$ units.

Note, here $\vec{p} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{q} = \hat{i} + 2\hat{j} + 3\hat{k}$.

14. (b) The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2 + c^2}$.

So, the required distance is $\sqrt{3^2 + 5^2} = \sqrt{34}$ units.

15. (d) $(2\vec{a}.\hat{i})\hat{i} - (\vec{b}.\hat{j})\hat{j} + (\vec{c}.\hat{k})\hat{k} = (2\times3)\hat{i} - (1)\hat{j} + (2)\hat{k} = 6\hat{i} - \hat{j} + 2\hat{k} = \vec{c}$.

16. (b) The points (1, 0) and (0, 2) satisfy the equation $2x + y = 2$ and shaded region shows that $(0, 0)$ doesn't lie in the feasible solution region. So, the inequality is $2x + y \ge 2$.

16. **FOR VISUALLY IMPAIRED**

(a) As (4, 0) and (0, 3) gives maximum value, so $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \quad \therefore 4a = 3b$.

17. (a) The student may read the point (2, 9) from the line on the graph. The student may find the equation $3x + y = 15$ joining (5, 0) and (0, 15) and then verify that the point (2, 9) satisfies it.

17. **FOR VISUALLY IMPAIRED**

(c) Open half plane that contains origin, but not the points of the line $3x+5y=10$.

18. (b) The person knows the first 4 digits. So, the person has to guess the remaining two digits.

$\therefore$ P(guessing the PIN) $=1\times1\times1\times1\times\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$.

19. (d) $\because \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\tan^{-1}1-\sec^{-1}(\sqrt{2})=\frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{4}=\frac{\pi}{3}\neq\frac{\pi}{4}$

So, Assertion is false.

Principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and that of $\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

So, Reason is true.

20. (c) As $\vec{r}\times(\vec{a}+\vec{b})=\vec{0}$ so, $\vec{r}$ is parallel to $(\vec{a}+\vec{b})$ and $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$.

So, $\vec{r}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$.

That is, $\vec{r}$ is perpendicular to $(\vec{a}\times\vec{b})$.

So, Assertion is true.

But $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$, so $(\vec{a}+\vec{b})$ is not perpendicular to the plane of $\vec{a}$ and $\vec{b}$.

Therefore, Reason is false.

SECTION B

21. (A) $\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$

$$=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$=\frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3-2\sqrt{3}+1}{3-1}=\frac{4-2\sqrt{3}}{2}.$$

$$=2-\sqrt{3}.$$

OR

(B) For domain of $\cos^{-1}(3x-2)$, we must have $-1\le 3x-2\le 1$

$-1+2\le 3x-2+2\le 1+2$

$\Rightarrow 1\le 3x\le 3$

$\Rightarrow \frac{1}{3}\le x\le 1$

So, domain of $\cos^{-1}(3x-2)$ is $x\in\left[\frac{1}{3},1\right]$.

22. Given $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Differentiating with respect to x, we get $\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\times\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\times\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \frac{1}{\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)} = \frac{1}{\sin 2\left(\frac{\pi}{4}+\frac{x}{2}\right)} = \frac{1}{\sin\left(\frac{\pi}{2}+x\right)} = \frac{1}{\cos x} = \sec x$$

$$\Rightarrow \frac{dy}{dx} - \sec x = 0 .$$

23. (A) $\int \frac{(x-3)e^x}{(x-1)^3} dx = \int \frac{(x-1-2)e^x}{(x-1)^3} dx$

$$= \int \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) e^x \, dx$$

$$= \int \left[\frac{1}{(x-1)^2} + \frac{d}{dx}\left(\frac{1}{(x-1)^2} \right) \right] e^x \, dx$$

$$= \frac{e^x}{(x-1)^2} + C \qquad \left[\because \int \{f(x) + f'(x)\} e^x dx = e^x f(x) + C \right.$$

OR

(B) Required area $= \text{ar(ONBO)} = \int_0^4 \sqrt{y} \, dy$

$$= \frac{2}{3}\left[y^{3/2} \right]_0^4$$

$$= \frac{2}{3}\left[4^{3/2} - 0^{3/2} \right]$$

$$= \frac{2}{3}\left[(4^{1/2})^3 - 0 \right]$$

$$= \frac{2}{3}\left[2^3 \right]$$

$$= \frac{16}{3} \text{ Sq. units} .$$

23. (B) **<u>FOR VISUALLY IMPAIRED</u>**

Required area $= \int_0^3 y \, dx$

$$= \int_0^3 \sqrt{x} \, dx$$

$$= \frac{2}{3}\left[x^{3/2} \right]_0^3$$

$$= \frac{2}{3}\left[3^{3/2} - 0^{3/2} \right]$$

$$= \frac{2}{3}\left[3\sqrt{3} \right]$$

$$= 2\sqrt{3} \text{ Sq. units} .$$

24. Given $f(x+y)=f(x)f(y)$

$\Rightarrow f(0+5)=f(0)f(5)$

$\Rightarrow f(5)=f(0)f(5)$

$\Rightarrow f(0)=1$

Now $f'(5)=\lim\limits_{h\to0}\dfrac{f(5+h)-f(5)}{h}=\lim\limits_{h\to0}\dfrac{f(5)f(h)-f(5)}{h}$ [$\because f(x+y)=f(x)f(y)$

$=\lim\limits_{h\to0}\dfrac{2f(h)-2}{h}$ [$\because f(5)=2$

$=2\lim\limits_{h\to0}\dfrac{f(h)-1}{h}=2\lim\limits_{h\to0}\dfrac{f(h)-f(0)}{h}=2\,f'(0)$

$=2(3)$ [$\because f'(0)=3$

$=6$

25. The vector $\overrightarrow{OP}=\dfrac{1}{2}(4\hat{i}+4\hat{k})=2\hat{i}+2\hat{k}$.

Area of the parallelogram formed by the two adjacent sides as OA and OP

$$=\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\2&0&2\end{vmatrix}$$

$=\left|2\hat{i}-2\hat{k}\right|$

$=\sqrt{2^2+0^2+(-2)^2}$

$=\sqrt{8}=2\sqrt{2}$ Sq. units.

SECTION C

26. (A) We have $x^y=e^{x-y}$

Taking log on both sides, we get $y\log x=(x-y)\log e$

$\Rightarrow y\log x=(x-y)\times1$ [$\because \log e=1$

$\Rightarrow y\log x+y=x$

$\Rightarrow y=\dfrac{x}{1+\log x}$

Differentiating with respect to x, we get $\dfrac{dy}{dx}=\dfrac{(1+\log x)\times1-x\times\frac{1}{x}}{(1+\log x)^2}$

$$\Rightarrow\frac{dy}{dx}=\frac{\log x}{(\log e+\log x)^2}$$

$$\Rightarrow\frac{dy}{dx}=\frac{\log x}{\{\log(x\,e)\}^2}$$

Now $\left.\dfrac{dy}{dx}\right]_{\text{at } x=e}=\dfrac{\log e}{(\log e^2)^2}=\dfrac{1}{(2\log e)^2}=\dfrac{1}{(2\times1)^2}=\dfrac{1}{4}$ [$\because \log e=1$

OR

(B) We have $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$

On differentiating w.r.t. θ, we get $\dfrac{dx}{d\theta}=a(1-\cos\theta),\ \dfrac{dy}{d\theta}=a(0+\sin\theta)=a\sin\theta$

$\because \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)}$

$\Rightarrow \dfrac{dy}{dx} = \cot\left(\dfrac{\theta}{2}\right)$

Now $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}\left[\cot\left(\dfrac{\theta}{2}\right)\right] = -\dfrac{1}{2}\text{cosec}^2\left(\dfrac{\theta}{2}\right)\times\dfrac{d\theta}{dx} = -\dfrac{1}{2}\text{cosec}^2\left(\dfrac{\theta}{2}\right)\times\dfrac{1}{a(1-\cos\theta)}$

$\Rightarrow \dfrac{d^2y}{dx^2} = -\dfrac{1}{2a}\text{cosec}^2\left(\dfrac{\theta}{2}\right)\times\dfrac{1}{2\sin^2\left(\frac{\theta}{2}\right)}$

$\therefore \dfrac{d^2y}{dx^2} = -\dfrac{1}{4a}\text{cosec}^4\left(\dfrac{\theta}{2}\right).$

27. Let r be the radius of ice ball at time t.

Volume, $V = \dfrac{4}{3}\pi r^3$ …(i)

And surface area, $S = 4\pi r^2$ …(ii)

Given $\dfrac{dV}{dt} \propto S$

$\Rightarrow \dfrac{dV}{dt} = -kS$ …(iii)

where k is some positive constant

Differentiating (i) w.r.t. t, we get $\dfrac{dV}{dt} = \dfrac{4}{3}\pi(3r^2)\times\dfrac{dr}{dt}$

$\Rightarrow \dfrac{dV}{dt} = 4\pi r^2\times\dfrac{dr}{dt}$ …(iv)

From (iii) and (iv), $-kS = 4\pi r^2\times\dfrac{dr}{dt}$

By (ii), $-kS = S\times\dfrac{dr}{dt}$

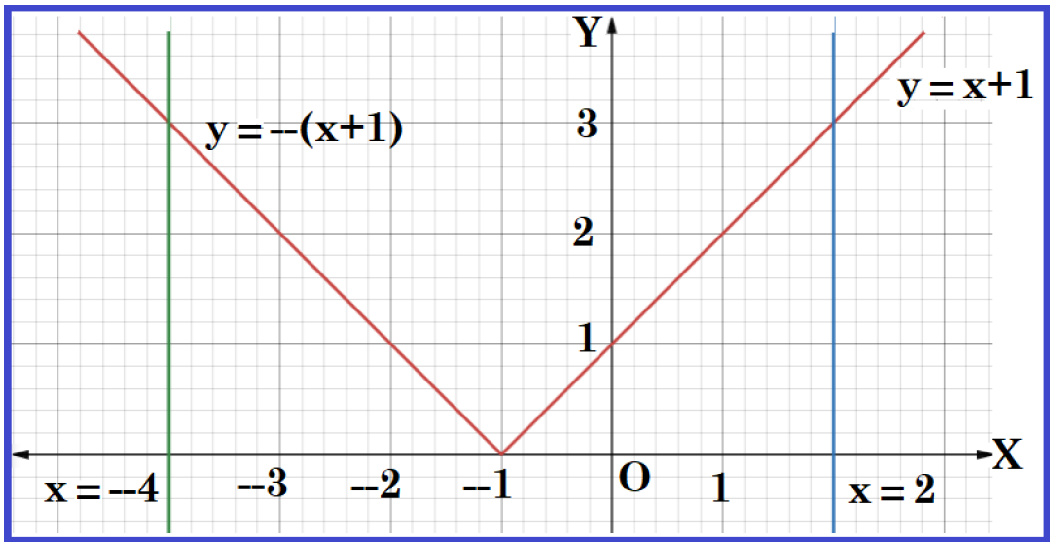
$\Rightarrow \dfrac{dr}{dt} = -k$

Thus, radius of the ice-ball decreases at a constant rate.

28. (A) We have $y = |x+1|$.

That is, $y = \begin{cases} x+1, \text{ if } x \geq -1 \\ -(x+1), \text{ if } x < -1 \end{cases}$.

Refer the graph of $y = |x+1|$ given below.



Now $\int_{-4}^{2}|x+1|dx=\int_{-4}^{-1}-(x+1)dx+\int_{-1}^{2}(x+1)dx$

$$=-\frac{(x+1)^2}{2}\Big]_{-4}^{-1}+\frac{(x+1)^2}{2}\Big]_{-1}^{2}$$

$$=-\left(0-\frac{9}{2}\right)+\left(\frac{9}{2}-0\right)=9.$$

This value represents the area of the region bounded by the curve $y=|x+1|$, x-axis and the lines $x=-4$ and $x=2$.

OR

(B) We have $\{(x, y): x^2-4y\le 0,\ y-x\le 0\}$.

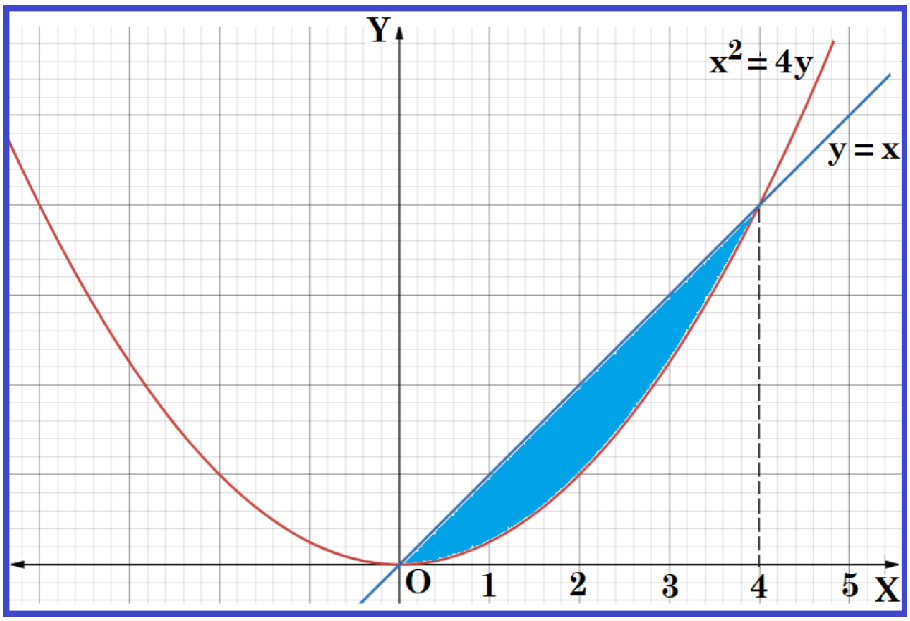
Consider $x^2-4y=0,\ y-x=0$.

On solving these equations, we get $x^2-4x=0$

$\Rightarrow x(x-4)=0$

$\Rightarrow x=0,\ x=4$

The points of intersection : (0, 0) and (4, 4).



$$\text{Required area} = \int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3}$$

$$= \frac{8}{3} \text{ Sq.units}.$$

28. **FOR VISUALLY IMPAIRED**

(A) We have $y = |x+1|$.

That is, $y = \begin{cases} x+1, \text{ if } x \geq -1 \\ -(x+1), \text{ if } x < -1 \end{cases}$.

Now $\int_{-4}^{2} |x+1|dx = \int_{-4}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx$

$$= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$$

$$= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9.$$

This value represents the area of shaded region bounded by the curve $y = |x+1|$, x-axis and the lines $x = -4$ and $x = 2$.

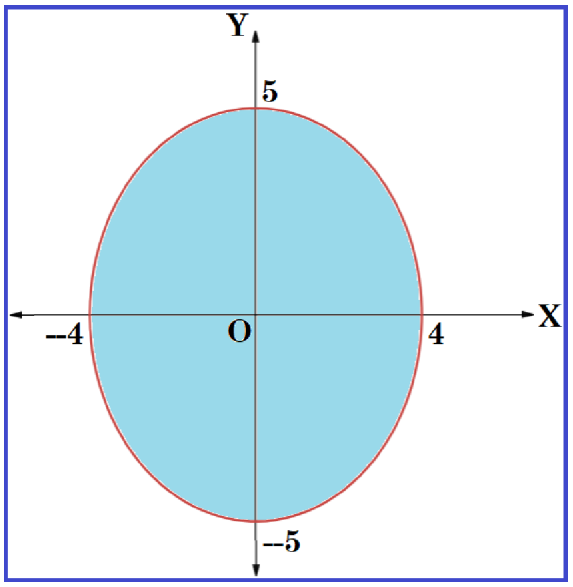
OR

(B) We have $25x^2 + 16y^2 = 400$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

$$\Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$$



$$\text{Required area} = 4\left[\int_0^4 \frac{5}{4}\sqrt{4^2 - x^2}dx\right]$$

$$= 5\left[\frac{x\sqrt{4^2 - x^2}}{2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$= 5\left[0 + 8\sin^{-1}(1) - 0\right]$$

$$= 40 \times \frac{\pi}{2}$$

$$= 20\pi \text{ Sq.units}.$$

29. (A) The line through (2, –1, 3) parallel to the z-axis is given by $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{k})$.

Any random point on this line is $P(2, -1, 3+\lambda)$.

Any point on the given line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \mu(3\hat{i} + 6\hat{j} + 2\hat{k})$ is $Q(2+3\mu, -1+6\mu, 2+2\mu)$.

For the intersection point, we must have $Q(2+3\mu, -1+6\mu, 2+2\mu) = P(2, -1, 3+\lambda)$

$\Rightarrow 2 = 2+3\mu$ i.e., $\mu = 0$ [On comparing the x-coordinates

The point of intersection is (2, –1, 2).

The distance from (2, –1, 3) to (2, –1, 2) is clearly 1 unit.

Note, we have used distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

OR

(B) The line through (2, –1, 1) parallel to the z-axis is $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{k})$.

Any random point on this line is $P(2, -1, 1+\lambda)$.

Any random point on the given line $\vec{r} = (3\hat{i} + \hat{k}) + \mu(\hat{i} + \hat{j} + \hat{k})$ is $A(3+\mu, \mu, 1+\mu)$.

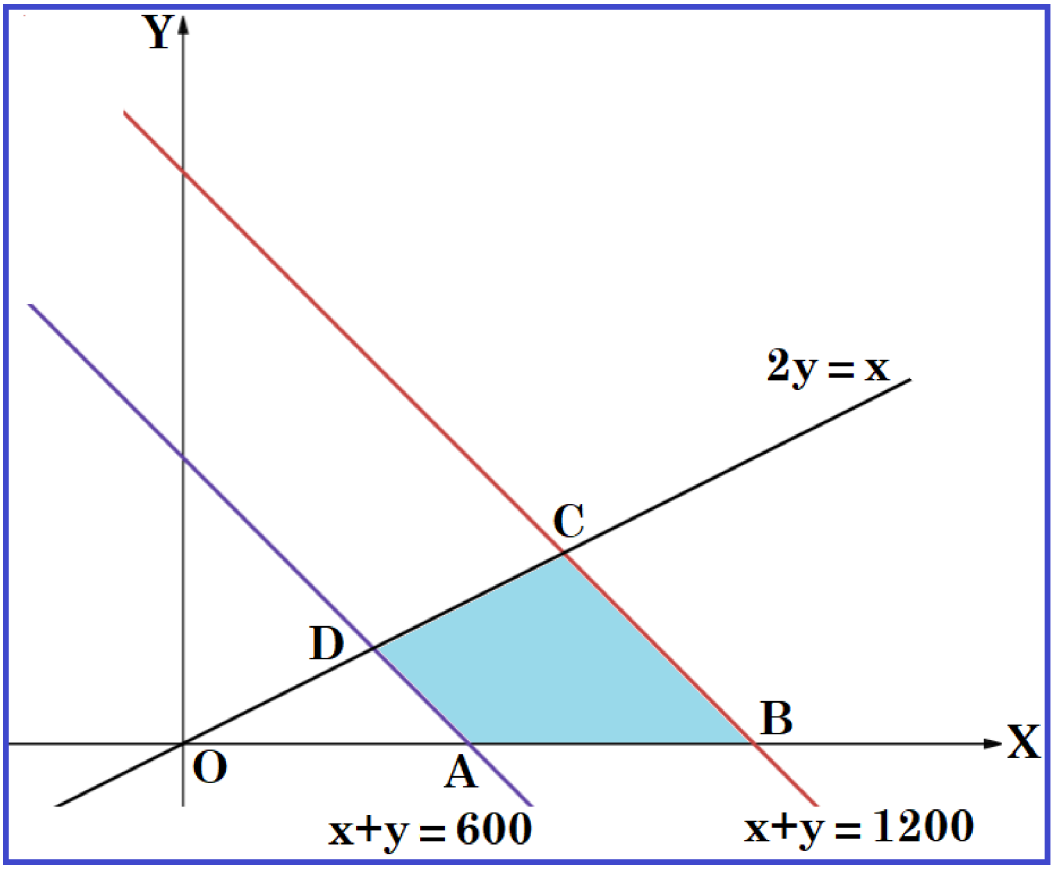
For the intersection point, we must have $A(3+\mu, \mu, 1+\mu) = P(2, -1, 1+\lambda)$

$\Rightarrow \mu = -1$ [On comparing the y-coordinates

Therefore, the point of intersection is (2, –1, 0).

The distance of (2, –1, 0) from the z-axis is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$ units.

30. Consider the graph shown.



Corner points are $A(600, 0), B(1200, 0), C(800, 400), D(400, 200)$.

Note that, $Z_A = 1200$, $Z_B = 2400$, $Z_C = 2000$, $Z_D = 1000$.

Clearly, the maximum value of $Z = 2400$ when $x = 1200$ and $y = 0$.

30. **FOR VISUALLY IMPAIRED**

At the given corner points $A(600, 0), B(1200, 0), C(800, 400)$ and $D(400, 200)$, the values of objective function $Z = 3x + 2y$ are $Z_A = 1800$, $Z_B = 3600$, $Z_C = 3200$ and $Z_D = 1600$.

Hence, the Maximum value of $Z = 3600$ at $B(1200, 0)$ and the minimum value of $Z = 1600$ at $D(400, 200)$.

31. Let the events be A : Mehul is selected and B : Rashi is selected.
Then according to the question, A and B are independent events.
Also $P(A) = 0.4,\ P(A \cap \overline{B}) + P(B \cap \overline{A}) = 0.5$.
Let $P(B) = x$.
Then $P(A \cap \overline{B}) + P(B \cap \overline{A}) = 0.5$
$\Rightarrow P(A)P(\overline{B}) + P(B)P(\overline{A}) = 0.5$
$\Rightarrow 0.4(1-x) + x(1-0.4) = 0.5$
$\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$
$\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$
$\Rightarrow x = \dfrac{0.1}{0.2} = \dfrac{1}{2} = 0.5$
So, probability of selection of Rashi $= 0.5$.
Probability of selection of at least one of them $= 1 - P(\overline{A} \cap \overline{B})$
$= 1 - P(\overline{A})P(\overline{B})$
$= 1 - 0.6 \times 0.5$
$= 1 - 0.3 = 0.7$.

SECTION D

32. $AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

So, $A^{-1} = B$ …(i)
Given system of equations is $3x - 6y - z = 3,\ 2x - 5y - z + 2 = 0,\ -2x + 4y + z = 5$.
In matrix form it can be written as $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$.
So, $X = A^{-1}C = BC$ [By (i), $A^{-1} = B$
$\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$
By equality of matrices, we get $x = 2,\ y = -3,\ z = 21$.

33. (A) Consider $I = \displaystyle\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$
Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$.
When $x = 0 \Rightarrow \theta = 0$ and when $x = 1 \Rightarrow \theta = \dfrac{\pi}{4}$.
$\therefore I = \displaystyle\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \times \sec^2\theta\, d\theta$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)d\theta$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\left[\frac{2}{1+\tan\theta}\right]d\theta$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log 2\,d\theta-\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)\,d\theta$$

$$\Rightarrow I=\log 2\left[\theta\right]_0^{\pi/4}-I$$

$$\Rightarrow 2I=\log 2\left[\frac{\pi}{4}-0\right]$$

$$\therefore I=\frac{\pi}{8}\log 2\,.$$

OR

(B) Let $I=\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta=\int\frac{(3\sin\theta-2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}d\theta$

Put $\sin\theta=t\Rightarrow\cos\theta\,d\theta=dt$

$$\therefore I=\int\frac{(3t-2)}{5-(1-t^2)-4t}dt$$

$$\Rightarrow I=\int\frac{(3t-2)}{t^2-4t+4}dt=\int\frac{3t-2}{(t-2)^2}dt$$

$$\Rightarrow I=\int\frac{(3t-6)+4}{(t-2)^2}dt$$

$$\Rightarrow I=\int\frac{3(t-2)+4}{(t-2)^2}dt$$

$$\Rightarrow I=\int\left[\frac{3(t-2)}{(t-2)^2}+\frac{4}{(t-2)^2}\right]dt$$

$$\Rightarrow I=\int\left[\frac{3}{(t-2)}+\frac{4}{(t-2)^2}\right]dt$$

$$\Rightarrow I=3\log|t-2|-\frac{4}{t-2}+C$$

$$\therefore I=3\log|\sin\theta-2|-\frac{4}{\sin\theta-2}+C$$

34. (A) $y+\frac{d}{dx}(xy)=x(\sin x+x)$

$$\Rightarrow y+\left(x\frac{dy}{dx}+y\right)=x(\sin x+x)$$

$$\Rightarrow 2y+x\frac{dy}{dx}=x(\sin x+x)$$

$$\Rightarrow \frac{dy}{dx}+\left(\frac{2}{x}\right)y=(\sin x+x)$$

This a linear differential equation of the form $\frac{dy}{dx}+P(x)\,y=Q(x)$.

Clearly, $P(x)=\frac{2}{x}$, $Q(x)=(\sin x+x)$.

$$\therefore \text{I.F.}=e^{\int\frac{2}{x}dx}=e^{2\log x}=e^{\log x^2}=x^2$$

Solution will be given by $y\times(\text{I.F.})=\int Q(x)\times(\text{I.F.})\,dx+C$

$$\Rightarrow yx^2=\int(\sin x+x)x^2\,dx+C$$

$$\Rightarrow yx^2=\int \sin x\times x^2dx+\int x^3\,dx+C$$

$$\Rightarrow yx^2=x^2\int\sin x\,dx-\int\left[\frac{d}{dx}(x^2)\int\sin x\,dx\right]+\frac{x^4}{4}+C$$

$$\Rightarrow yx^2=-x^2\cos x+2\int x\cos x\,dx+\frac{x^4}{4}+C$$

$$\Rightarrow yx^2=-x^2\cos x+2\left\{x\int\cos x\,dx-\int\left[\frac{d}{dx}(x)\int\cos x\,dx\right]dx\right\}+\frac{x^4}{4}+C$$

$\therefore yx^2=-x^2\cos x+2\{x\sin x+\cos x\}+\frac{x^4}{4}+C$, which is the required solution.

OR

(B) $2ye^{x/y}dx+(y-2xe^{x/y})\,dy=0$

$$\Rightarrow \frac{dx}{dy}=\frac{2xe^{x/y}-y}{2ye^{x/y}}=\frac{2\frac{x}{y}e^{\frac{x}{y}}-1}{2e^{\frac{x}{y}}}$$

It is a homogeneous differential equation, as $\frac{dx}{dy}=f\left(\frac{x}{y}\right)$.

To solve, put $x=vy\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$

Therefore, $v+y\frac{dv}{dy}=\frac{2ve^v-1}{2e^v}$

$$\Rightarrow y\frac{dv}{dy}=\frac{2ve^v-1}{2e^v}-v=\frac{2ve^v-1-2ve^v}{2e^v}$$

$$\Rightarrow y\frac{dv}{dy}=\frac{-1}{2e^v}$$

$$\Rightarrow 2e^vdv=-\frac{dy}{y}$$

On integrating, $\int 2e^{v}dv = -\int \frac{dy}{y}$

$\Rightarrow 2e^{v} = -\log|y| + C$

$\Rightarrow 2e^{\frac{x}{y}} + \log|y| = C$

Since $y(0) = 1$ i.e., when $x = 0,\ y = 1$.

Clearly $2e^{0} + \log|1| = C$ i.e., $C = 2$ [$\because \log 1 = 0$

Hence, the required solution is $2e^{\frac{x}{y}} + \log|y| = 2$.

35. Rewriting the line $\frac{x-1}{3} = -y,\ z+1 = 0$, we get $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda$ (say).

Any random point on it is $(3\lambda+1, -\lambda, -1)$.

For the point where y-coordinate is 1, we have $-\lambda = 1 \Rightarrow \lambda = -1$.

Therefore, the point is (–2, 1, –1).

Let the direction ratios of the required line be a, b, c.

Also for the given lines $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0}$ and $\frac{x}{-2} = \frac{y+1}{2} = \frac{z+2}{1}$, the direction ratios are respectively 3, –1, 0 and –2, 2, 1.

Then using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get $3a - b = 0$ and $-2a + 2b + c = 0$

On solving, we get $\frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$.

Hence, the required line passing through (–2, 1, –1) is given by $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4}$.

In the vector form, the equation of the required line is $\vec{r} = (-2\hat{i} + \hat{j} - \hat{k}) + \mu(-\hat{i} - 3\hat{j} + 4\hat{k})$.

SECTION E

36. **(i)** Traffic flow is not reflexive as $(A, A) \notin R$.

(ii) Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R$, but $(A, E) \notin R$.

(iii) (A) $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$

Clearly, the Domain $= \{A, B, C, D\}$ and the Range $= \{B, C, D, E\}$.

OR

(iii) (B) No, the traffic flow doesn't represent a function as A has three images.

Note that, $(A, B), (A, C), (A, D) \in R$.

37. **(i)** $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150) = -0.8x^2 + 30x - 150$.

(ii) For $P(x) = -0.8x^2 + 30x - 150$, $P'(x) = -1.6x + 30$

For critical points, $P'(x) = 0$ i.e., $-1.6x + 30 = 0$

$$\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75.$$

(iii) (A) Note that, $P''(x) = -1.6$.

As $P''(18.75) = -1.6 < 0$.

So, critical value $x = 18.75$ corresponds to a maximum profit.

OR

(iii) (B) As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit.

Also, the Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150 = -259.2 + 540 - 150 = 130.80$.

Therefore, the maximum profit is ₹130.80.

38. Let the events be given as

E_1 : the students is in the first group (time spent on screen is more than 4 hours)

E_2 : the student is in the second group (time spent on screen is 2 to 4 hours)

E_3 : the student is in third group (time spent on screen is less than 2 hours)

A : the event of the student showing symptoms of anxiety and low retention.

So, $P(E_1)=\frac{60}{100}$, $P(E_2)=\frac{30}{100}$ and $P(E_3)=\frac{10}{100}$.

Also $P(A|E_1)=\frac{80}{100}$, $P(A|E_2)=\frac{70}{100}$ and $P(A|E_3)=\frac{30}{100}$.

(i) Using total probability theorem, $P(A)=P(E_1)\times P(A|E_1)+P(E_2)\times P(A|E_2)+P(E_3)\times P(A|E_3)$

$\Rightarrow P(A)=\frac{60}{100}\times\frac{80}{100}+\frac{30}{100}\times\frac{70}{100}+\frac{10}{100}\times\frac{30}{100}=\frac{72}{100}$

$\therefore P(A)=72\%$.

(ii) Using Bayes' theorem, $P(E_1|A)=\frac{P(E_1\cap A)}{P(A)}=\frac{P(E_1)\times P(A\,|\,E_1)}{P(A)}$

$\Rightarrow P(E_1|A)=\frac{\frac{60}{100}\times\frac{80}{100}}{\frac{72}{100}}=\frac{48}{72}$

$\therefore P(E_1|A)=\frac{2}{3}$.